Clock synchronization with a three clock system

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Abstract

Observed from an inertial frame *K*, a moving clock *m* appears to run slow. However, a co-moving inertial frame, attached with the moving clock, observes that the clocks in *K* are asynchronous as well as slowing down. The transformation of event coordinates from *K* to *M* and the inverse of that transformation are symmetric and in fact identical if we reverse the direction of the spatial coordinate axis. This implies that *K* and *M* are equivalent and therefore the clocks in *K* and *M* are also equivalent. In this paper we attempt to synchronize two clocks *m* and *n* in relative motion using a third clock *k,* and test the synchronicity of *m* and *n* when they meet. The clocks can also be imagined as intersecting traces on the space-time continuum.

Introduction

Time dilation is the phenomenon where the observed time rate of an observer's reference frame is different from that of a different reference frame. In special relativity, clocks that are moving with speed *v* with respect to an inertial system of observations are found to be running slower. Einstein [1] showed that the Lorentz transformation equations describe the relation between (x, y, z, t) and (x', y', z', t') under the principles of equivalence of inertial frames and constancy of the speed of light. The Lorentz transformation equations, according to the principle of relativity, describe the relationship between event coordinates observed by inertial frames in uniform relative motion. Under these transformations an object of proper length L_0, appears to have contracted to a length L as seen from another inertial frame; the relationship between L and L_0 is given by the formula

$$L = L_0 / \gamma$$

where $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$ and *c* is the speed of light in vacuum. Thus in a co-moving coordinate system, the length L' will still appear to be equal to L_0.

The formula for determining time dilation in special relativity is:

$\Delta t = \gamma\, \Delta t_0$ where
Δt_0 is a time interval as measured with a "moving" clock,
Δt is that same time interval as measured in a "stationary" system,
v is the relative speed between the clock and the stationary system, and
c is the speed of light.

Thus the Lorentz transformations of spatial and temporal event coordinates between two inertial frames in relative motion ordain that a particular clock of one frame observed from another frame appears to run slow, and the set of clocks in one frame appears asynchronous as well as slowing down when viewed from the other frame. The asynchronicity and the slowing down seem to combine to create a symmetric perception of each other's frame.

The Lorentz transformations are symmetric so that any two inertial frames appear identical. The inverse of the transformation is identical with the parameter v replaced with $-v$, and in fact the inverse transformation is identical to the forward transformation if we reverse the positive direction of the spatial axis – in this case there is no change in the sign of the parameter 'v'. Thus both frames see each other moving at a velocity 'v' in opposite directions. Since the choice of the positive direction of the spatial axis is arbitrary, the two inertial frames are or appear to be identical under the Lorentz transformations.

The question whether a moving clock runs slow or only appears to run slow is an intriguing one. For all practical purposes a moving clock runs slow. However, if an observer is attached to the moving clock, his perception will be that the set of clocks in the inertial frame that is observing him are asynchronous and for this reason he concludes that the moving clock is slowing down. For the observer attached to the moving clock, the rate at which his clock is running is indeed the 'correct' rate, and any conclusion to the contrary is due to improper synchronization, which indeed is the result of the slowing down of the clocks that are 'moving,' according to his perception. According to [2], there is no difficulty in synchronizing two clocks in the same frame of reference; only when a clock is moving relative to a given frame of reference do ambiguities of synchronization or simultaneity arise.

In this paper we propose a constructive procedure for synchronizing a three-clock system. We assume that all clocks (even if in relative motion) run at the same rate. All the three clocks are under uniform relative motion in relation to each other and each one of them falls strictly under the purview of special relativity and the Lorentz transformations. From our procedure one can conclude that clocks in relative motion do not run identically.

1. The three clock system

We describe a three-clock system from some arbitrary inertial frame in the following fashion. Three identical clocks k, m and n are in relative motion with velocities v, u, and w, and at some instant appear as below:

$k \rightarrow v$ $\qquad\qquad\qquad$ $m \rightarrow u$ $\qquad\qquad\qquad$ $n \rightarrow w$

such that $v > u > w$. Furthermore we assume that the spatial separation of the clocks are such that the events E_1 (k passing m), E_2 (k passing n), and E_3 (m passing n) happen in the order E_1, E_2, E_3. We design our thought experiment so that when E_1 occurs (that is, when k and m pass each other), m synchronizes its clock with k; similarly when E_2 occurs (that is, when k and n pass each other), n synchronizes its clock with k. Thus we presume that after the event E_2, both clocks m and n are also synchronized as they are both synchronized with clock k.

We would like to examine the correctness of this presumption by applying the Lorentz transformations and in particular by the actual observations of m and n as they pass each other at the occurrence of event E_3.

For simplicity we take our inertial reference frame to be the co-moving frame N attached with clock n. Thus we have $w = 0$. And we take the relative velocity between clocks k and n as 'v' and the relative velocity between clocks m and n as 'u.'

However, when m and n meet (event E_3), we find that they do not show the same time, even though both had synchronized themselves with clock k at events E_1 (k and m) and at event E_2 (k and n).

2. Analysis of time dilation

Let us say that event E_1 occurs at a distance of s from clock n (in frame N). Therefore, clock k will reach clock n after a time of (s/v). However, clock k will show a time of

$(s/v)\sqrt{1-\frac{v^2}{c^2}}$ when it reaches clock n due to time dilation. According to the procedure set out in our thought experiment we synchronize clock n with clock k when they meet at event E_1. Therefore, at event E_2 both clocks k and n show a time of $(s/v)\sqrt{1-\frac{v^2}{c^2}}$.

According to frame N at this time m would have traveled a distance $u(s/v)$ and the distance left for m to reach n is $s - u(s/v)$. This distance will be covered in a time interval of $(s/u) - (s/v)$. This time will be clocked by n between E_2 and E_3, and thus at E_3 clock n will read

$$[(s/u) - (s/v)] + [(s/v)\sqrt{1-\frac{v^2}{c^2}}].$$

When m reaches n, clock m will read $(s/u)\sqrt{1-\frac{u^2}{c^2}}$.

Thus the difference between clocks m and n when they meet at the occurrence of event E_3 will be

$$[(s/u) - (s/v)] + [(s/v)\sqrt{1-\frac{v^2}{c^2}}] - [(s/u)\sqrt{1-\frac{u^2}{c^2}}].$$

It is easily seen that this difference is independent of any inertial frame chosen for the analysis; either K, M, N or any arbitrary inertial frame. This is so because this time difference is recorded by clocks m and n at the same space-time point that is event E_3, when both clocks m and n are physically present at the same place at the same time. This difference is not zero.

Further, as a special case we can let $u \rightarrow 0$, and after taking appropriate limits, the above expression becomes

$$(s/v)\,[\sqrt{1-\frac{v^2}{c^2}} - 1].$$

The events E_1, E_2 and E_3 described as above are observed in all the three frames K, M and N. It is also true that the time intervals $E_1 \sim E_2$ and $E_2 \sim E_3$ add up to give the time interval $E_1 \sim E_3$ in all the three frames K, M and N. The time interval $E_1 \sim E_2$ is clocked lowest by frame K as clock k is present at both the space-time points E_1 and E_2.

Similarly, the time interval $E_2 \sim E_3$ is clocked lowest by N and time interval $E_1 \sim E_3$ is clocked lowest by M. The procedure for our synchronization experiment gives the final difference between clocks m and n as

$(E_1{\sim}E_2)_K + (E_2{\sim}E_3)_N – (E_1{\sim}E_3)_M$

The above quantity would have been zero if the reference frame was same for the three time intervals. It would be zero under Galilean/Newtonian considerations even retaining the different subscripts. However, under relativistic considerations it is not zero. When two events occur at the same spatial location in an inertial frame, the time interval between those two events is observed to be larger in any other inertial frame.

Since we have specified the velocities of *K* and *M* with respect to *N* as *v* and *u* respectively, it was convenient to base our reference frame as *N* to arrive at the time difference between clocks *m* and *n*. If we base our considerations from any arbitrary frame instead of frame *N*, then by using the relativistic velocity addition formulae, we can show that the above expression remains the same in value; this is as it should be because this is the difference observed by clocks *n* and *m* at the same space-time point E_3 and any observation at the same space-time point is independent of the reference frame.

3. Discussion

The difference in time shown by clocks *m* and *n* when they meet can be explained by assuming any one of the following statements:

1. Frame *K* is stationary and isotropic. Clocks *m* and *n* run slow with respect to *K*.
2. Frame *M* is stationary and isotropic. Clocks *k* and *n* run slow with respect to *M*.
3. Frame *N* is stationary and isotropic. Clocks *k* and *m* run slow with respect to *N*.
4. Any other arbitrary inertial reference frame *P* is stationary and isotropic. Clocks *k*, *m,* and *n* run slow with respect to *P* as a function of their velocities.

We observe that in none of the above scenarios do clocks *k*, *m,* and *n* run identically. So we may conclude that clocks in relative motion do not run identically. There are two possible consequences of this result. One possibility is that there exists a unique isotropic 'stationary' reference frame *S*, in relation to which physical processes and clocks run slow in all other inertial frames (which are in relative motion with respect to *S*).

The other possibility is that clocks *k*, *m* and n are traces on the space-time continuum. The three events E_1, E_2 and E_3 are the intersection of these traces (like vertices of a triangle). This possibility visualizes any particular existence of a clock *k*, *m* or *n* at a space-time point as a permanent etching on the space-time continuum. Here the temporal sequences are only an interpretation of a particular inertial frame and in the space-time continuum there is no specific sequence, either temporal or spatial.

References

[1] Einstein A *Relativity, The Special and General Theory*, Authorized Translation by Robert W. Lawson, Three Rivers Press, New York, 1961.
[2] Sears F, Zemansky M, and Young H *University Physics,* Addison-Wesley, 1980.