The One-Way Speed of Light: A Simple Formulation

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Abstract

The Michelson-Morley experiment determined that the round-trip speed of light in any inertial frame is a constant and is equal to the universal constant *c* in all inertial fames. Normally a synchronization convention is chosen, as proposed by Einstein, which forces the one-way speed of light to be *c* in all directions in all inertial frames. We propose a unique formulation for the one-way speed of light in three-dimensional Euclidean space that is consistent with the Michelson-Morley experiment as well as the IST transformation, which is the Lorentz transformation without the resynchronization component. This formulation of a variable one-way speed of light is the only possible prescription that will satisfy the requirement that a synchronization shift along the line of relative motion between the chosen inertial frame and the rest frame will render the one-way speed of light to be a constant. Furthermore, this formulation satisfies the requirement that the harmonic mean of the onward and return speeds along any given line is *c* and the average speed along any closed path is also equal to *c*.

1 Introduction

It is well known that the Michelson-Morley experiment proves that the round-trip speed of light is a constant in any inertial frame [1]. The Einsteinian synchronization convention in any inertial frame fixes the one-way speed of light as constant [2, 3, 4]. There has been some debate by many authors [2, 3, 4, 5, 6] whether such a convention is really required or not.

In this paper we propose a unique formulation $C(\phi, \theta) = c/[1+(v/c)\sin\phi\cos\theta]$ for the one-way speed of light in 3-dimensional Euclidean space that is consistent with the Michelson-Morley experiment and the IST transformation [5]. This result can also be derived through a constructive approach by considering relative motion, Lorentz length contraction, and time dilation [7].

2 Formulation of one-way speed of light

It is well known [5, 6] that the Lorentz transformation can be decomposed into an IST transformation and a resynchronization component as indicated in equation (1).

$$\begin{bmatrix} \gamma & -v\gamma \\ -v\gamma/c^2 & \gamma \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ \frac{-v}{c^2} & 1 \end{bmatrix}}_{\text{Resynchronization}} \quad \underbrace{\begin{bmatrix} \gamma & -v\gamma \\ 0 & 1/\gamma \end{bmatrix}}_{\text{IST Transformation}} \tag{1}$$

The main contention of some authors [3, 4, 5] is that the resynchronization part is unnecessary and the IST part still retains the two-way speed of light to be a constant. We formulate a variable one-way speed of light consistent with the invariable two-way speed of light as follows.

Consider an Euclidean 3-dimensional space with a line making angle ϕ with the z-axis $(0 \le \phi \le \pi)$. The projection on the *x y* plane makes an angle θ with the positive *x*-axis. The standard formulation for the Cartesian coordinates for any point at a distance *r* from the origin is

$$x = r \sin\phi \cos\theta \tag{2a}$$

$$y = r \sin\phi \sin\theta \tag{2b}$$

$$z = r \cos\phi \tag{2c}$$

The condition $(0 \le \phi \le \pi)$ makes the projection on the *x y* plane $r \sin\phi$ always positive.

Supposing a light ray is propagating along this line designated by ϕ and θ with a variable propagation speed of $C(\phi, \theta)$; then at any instant *t* it generates an event (*x*, *y*, *z*, *t*):

$$x = C(\phi, \theta) \sin\phi \cos\theta\, t \tag{3a}$$

$$y = C(\phi, \theta) \sin\phi \sin\theta\, t \tag{3b}$$

$$z = C(\phi, \theta) \cos\phi\, t \tag{3c}$$

$$t = t$$

This event should transform by a resynchronization process as discussed in equation (1) to a propagation at speed *c*. Therefore we have

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{-v}{c^2} & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} C(\phi,\theta) * t * \sin\phi * \cos\theta \\ C(\phi,\theta) * t * \sin\phi * \sin\theta \\ C(\phi,\theta) * t * \cos\phi \\ t \end{bmatrix} = \begin{bmatrix} ct'\sin\phi\cos\theta \\ ct'\sin\phi\sin\theta \\ ct'\cos\phi \\ t' \end{bmatrix} \tag{4}$$

(Note: The line of relative motion between the inertial frame under consideration and the “rest frame” is taken to be the *x*-axis for simplicity. This can be generalized by appropriate spatial rotations.)

Equation (4) can be written out as follows.

$$C(\phi,\theta)\ t\sin\phi\cos\theta = ct'\sin\phi\cos\theta \Rightarrow C(\phi,\theta)t = ct'$$

$$C(\phi,\theta)\ t\sin\phi\sin\theta = ct'\sin\phi\sin\theta \Rightarrow C(\phi,\theta)t = ct'$$

$$C(\phi,\theta)\ t\cos\phi = ct'\cos\phi \Rightarrow C(\phi,\theta)t = ct'$$

$$C(\phi,\theta)\ t\sin\phi\cos\theta(-v/c^2) + t = t'$$

From the last equation we get,

$$\frac{t'}{t} = 1 - \frac{v}{c^2}\sin\phi(\cos\theta) * C(\phi,\theta)$$

Substituting this into any one of the other three equations, we obtain

$$C(\phi,\theta) = c\left(\frac{t'}{t}\right) = c - \frac{v}{c}\sin\phi\cos\theta * C(\phi,\theta)$$

$$C(\phi,\theta) = c - \frac{v}{c}\sin\phi\cos\theta * C(\phi,\theta)$$

$$\therefore C(\phi,\theta)\left[1 + \frac{v}{c}\sin\phi\cos\theta\right] = c$$

$$C(\phi,\theta) = \frac{c}{1 + \frac{v}{c}\sin\phi\cos\theta} \qquad (5)$$

The return speed by this formulation is

$$C^{-}(\phi,\theta) = \frac{c}{1 + \frac{v}{c}\sin(\pi-\phi)\cos(\pi-\theta)}$$
$$= \frac{c}{1 - \frac{v}{c}\sin\phi\cos\theta} \qquad (6)$$

It follows easily from equations (5) and (6) that the harmonic mean of these two speeds is *c*. It can also be shown by considering a closed path divided into small straight line elements with appropriate reflecting mirrors that the average round-trip speed of light will be *c* when the propagation is according to this formulation (see Appendix A).

3 Conclusion

We have provided a simple formulation for the one-way speed of light in any inertial frame and this formulation is consistent with universal synchronization and constancy of the two-way speed of light. In a recent paper [7], we have shown that this formulation can also be derived from Lorentz length contraction and time dilation.

References

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Appendix A

Consider any one straight-line element of length ΔL with angle ϕ with *z*-axis and the projection on the *x y* plane making angle θ with the *x*-axis. The light ray will take a time Δ*t* to traverse this element and the relation between them will be

$$\Delta t = \frac{\Delta L}{C(\phi,\theta)}\text{; where } C(\phi,\theta) = \frac{c}{1+\frac{v}{c}\sin\phi\cos\theta}$$

$$= \frac{\Delta L}{c}\ [1+\frac{v}{c}\sin\phi\cos\theta\ ]$$

$$= \frac{\Delta L}{c} + \frac{v}{c^2}\Delta L\sin\phi\cos\theta$$

We remark that the quantity ($\Delta L\sin\phi\cos\theta$) = Δ*x*.

When we sum over a path consisting of many elements, the sum of the second terms in the above equation becomes $\frac{v}{c^2}$ ΣΔ x. When this path is a closed path with ΣΔx = 0, the total time taken for a closed path is

$$\Sigma\frac{\Delta L}{c} = \frac{1}{c}\Sigma\Delta L = \frac{L}{c}.$$

Therefore the average speed of light over a closed path is equal to *c*.

What this means is that whenever light traverses a length element Δ*L*, it takes a time interval that has two components. One is (Δ*L*/*c*), and the other is (Δ*x*/*c*). The first component is always positive, as the Euclidean distance Δ*L* is always positive. The second component (Δ*x*/*c*) can be positive or negative and over a closed path the sum of the second component becomes zero.